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DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS  
SCHOOL OF ENGINEERING  
OLD DOMINION UNIVERSITY  
NORFOLK, VIRGINIA

DEVELOPMENT OF A NONLINEAR VORTEX METHOD

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Osama A. Kandil, Principal Investigator

Progress Report  
For the period October 1, 1980 - March 31, 1981

*Prepared for the*  
National Aeronautics and Space Administration  
Langley Research Center  
Hampton, Virginia

*Under*  
Research Grant NSG 1560  
E. Carson Yates, Jr., Technical Monitor  
Structures and Dynamics Division

May 1981



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SCHOOL OF ENGINEERING  
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*Submitted by the*  
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P.O. Box 6369  
Norfolk, Virginia 23508



May 1981

## DEVELOPMENT OF A NONLINEAR VORTEX METHOD

By

Osama A. Kandil\*

### I. INTRODUCTION

The present semi-annual report includes the progress of the research work conducted under this grant toward the development of reliable nonlinear vortex methods for predicting the steady and unsteady aerodynamic loads of highly sweptback wings at large angles of attack. It also presents abstracts of the papers, talks, and theses produced through this work. This report covers the period from October 1, 1980 to March 31, 1981.

During this period, research work has been concentrated on developing the following two methods:

1. Modified Nonlinear Discrete Vortex (MNDV) method.
2. Nonlinear Hybrid Vortex (NHV) method.

The first method is a modified version of the NDV-method where a realistic model of the inviscid leading-edge vortex core is introduced by the principal investigator. The modified method alleviates the problems previously encountered in predicting satisfactory pressure distributions on highly sweptback wings. Moreover, it enjoys a remarkable success in predicting, for the first time, the latest experimental data published by Hummel.<sup>1</sup>

Although the old NDV-method was almost abandoned in predicting the flow details and the distributed aerodynamic characteristics of this problem, the modified NDV-method pinpointed and cured the causes of problems encountered with the old method. The preliminary published results of the MNDV-method were well received. In this regard, the P.I. has received two letters of recognition, the first from

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\* Associate Professor, Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23508.

<sup>1</sup> Hummel, D., "On Vortex Formation Over a Slender Wing at Large Angles of Incidence," AGARD CP-247, January 1979.

Professor Hummel of the Institute of Fluid Mechanics of the Technical University in Braunschweig, West Germany and the second from Dr. Nargia of the Department of Aeronautical Engineering of the University of Bristol, England. Copies of these letters are enclosed in Appendix A.

The results obtained from the NHV-method for wings having side-edge separations have shown that the calculated spanwise variations of the load coefficients are in good agreement with the experimental data of Scholz.<sup>2</sup> Comparisons of the results with those of the NDV-method have shown that the NHV-method requires less number of vortex panels than the latter method for the same accuracy.

In the next two sections, specific points of progress and abstracts of papers are presented for each method.

## II. MODIFIED NONLINEAR DISCRETE VORTEX (MNDV) METHOD

1. A paper describing this method was presented at the 12th Congress of the International Council of the Aeronautical Sciences, Munich, Federal Republic of Germany, October 12-17, 1980. The paper is titled, "Numerical Prediction of Vortex Cores from the Leading and Trailing Edges of Delta Wings." The paper is attached as Appendix B.
2. A preliminary version of this paper was also presented at the 28th Meeting of the Aerospace Flutter and Dynamics Council, Williamsburg, VA, Oct. 1-3, 1980.
3. More results of this method for 3/4 turn of the leading-edge vortex system were presented at the 17th Annual Meeting of the Society of Engineering Sciences, Atlanta, Georgia, Dec. 15-17, 1980. The paper is titled, "Modeling of Vortex Cores and Feeding Sheets of Delta Wings using the Nonlinear Discrete Vortex Technique." The published abstract is attached as Appendix C.

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<sup>2</sup> Scholz, V.N., "Kraft and Druckverteilungsmessungen an Tragflächen Kleiner Streckung," *Forscharb. Ing. Wes.*, No. 16, pp. 85-92.

4. Currently the technique is modified for accurate near-field calculations by replacing the concentrated vortex segments with vortex panels having linear vorticity distributions. The vorticity functions are expressed in terms of the unknown circulations of the original concentrated vortex segments. This is opposite to what is being done with the NHV-method. For this purpose, the recently developed and highly efficient computer code of the velocity field will be used. Applications of this final version of the MNDV-method to delta wings with various aspect ratios and various angles of attack will be performed to study the effects of those two parameters on the formation and interaction of the vortex cores.

These results will be presented as a part of the AIAA paper No. 81-1263 to be presented at the AIAA 14th Fluid and Plasma Dynamics Conference, Palo Alto, California, June 23-25, 1981. The paper is titled, "Recent Improvements in the Prediction of the Leading and Trailing Edge Vortex Cores of Delta Wings." The other part of the paper deals with the viscous modeling of the vortex cores. This work is supported under a separate contract with the Naval Air Development Center, Warminster, PA. A copy of the paper abstract is attached as Appendix D.

### III. NONLINEAR HYBRID VORTEX (NHV) METHOD

1. The M.S. thesis of Mr. Li-Chuan Chu is completed in December, 1980. The thesis is titled, "A Nonlinear Hybrid Vortex Method for Wings Having Side-Edge Separations." The abstract of the thesis is attached as Appendix E and a copy of the thesis was delivered to the technical monitor, Dr. E. Carson Yates, Jr., who also served as a member of the thesis committee of Mr. Chu.

Currently, Mr. Chu is assisting in the development of the NHV-method for leading- and side-edge separations for steady-flow problems and for unsteady-flow problems due to small oscillations of wings around large mean angles of attack. These problems form substantial portions of his Ph.D. dissertation.

2. The results of the M.S. thesis were presented at the 28th Meeting of the Aerospace Flutter and Dynamics Council, NASA-Langley Research Center, Hampton, VA, Oct. 3, 1980.
3. A highly efficient computer code for the exact calculation of the velocity field induced by triangular and quadrilateral vortex panels having linear vorticity distribution has been developed, tested, and completed. Work is underway to develop the necessary conditions to eliminate the singularity of the normal component of the induced velocity along the panel edges. It is a logarithmic singularity arising from the geometric discretization of the vortex sheet. Mr. Thomas Tureaud, M.S. student, is assisting in this problem for his M.S. thesis. He will also be working on converting the steady-flow computer code from the CYBER-175 computer to the CYBER-203 computer.
4. The steady-flow computer code for leading- and side-edge separations which employs triangular vortex panels on the free-vortex sheets is completed. Currently, it is tested for rectangular, delta, and clipped-delta wings.
5. The formulation of the unsteady-flow problem using the NHV-method is completed. The computer program developed in Item 4 will be extended and modified for the unsteady-flow problem.

Due to the amount of work done this year, due to the complexity and importance of the present problems, and due to the natural lack of experience of some of the beginning students, we have used the computer relatively heavily in the past 6-12 months. Certain measures are now taken to closely monitor the computer usage.

APPENDIX A



PROF. DR.-ING DIETRICH HUMMEL  
IM INSTITUT FÜR STRÖMUNGSMECHANIK  
TECHNISCHE UNIVERSITÄT BRAUNSCHWEIG

3300 BRAUNSCHWEIG Dec. 10,  
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TELEFON: (0531) 323690 UND 3912433

H1/Ho/80/204

Dr. Osama A. Kandil  
Associate Professor  
Dept. of Mechn. Engineering and  
Mechanics  
Old Dominion University  
Norfolk, VA 23508  
- U.S.A. -

Subject: Vortex formation behind delta wings

Reference: Your paper, presented at the 12th ICAS Congress in Munich, 1980

Dear Dr. Kandil,

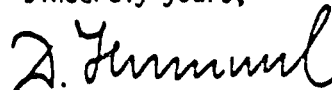
I thank you very much for having sent me a preprint-copy of your paper presented at the ICAS-Congress in Munich. I have read your paper with great interest and I congratulate you for your excellent results.

Unfortunately, I was not able to attend the ICAS-Congress because I was abroad. I spent about two months in Korea to deliver a lecture series there. Therefore, I missed your lecture in Munich as well as many other interesting papers. But I heard that there was a good discussion after your talk.

This is to let you know that we are presently engaged in similar experiments on double-delta wings where we get a twin-vortex system on each side of the wing. In addition, a wing-canard model is under construction which will enable us in near future to measure the vortex system of such a configuration. We will publish results as soon as possible and we hope that you will perform the corresponding calculations using your program.

With best wishes for a merry christmas and a happy new year I am

sincerely yours,



(Prof. Dr.-Ing. D. Hummel)

UNIVERSITY OF BRISTOL  
DEPARTMENT OF AERONAUTICAL ENGINEERING

Professor of Aeronautical Engineering:  
L. F. CRABTREE, Ph.D., C.Eng., F.R.Ae.S.

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Professor O.A. Kandil,  
Mechanical Engineering and Mechanics Department,  
Old Dominion University,  
Norfolk,  
Va. 23508  
U.S.A.

20th November, 198

Dear Professor Kandil,

Vortex Flow Reports

As promised, I have pleasure in sending copies of the following reports. You might care to simulate some of the flows with vortex sheet models particularly on wings with camber at low angles of attack.

- Nangia, R.K. A Study of Slender, Thin, Conically Cambered Wings with Flow Separation.  
University of Bristol, Department of Aeronautical Engineering.  
Report RKN/7701 (October 1977). Revised 1980.
- Nangia, R.K. Slender, Thick, Sharp-Edged Conically Cambered Wings with Flow Separation.  
University of Bristol, Department of Aeronautical Engineering.  
Report RKN/7801 (June 1978).
- Nangia, R.K. A Study of Slender Conical Thick Wings and Bodies with Variation of Flow Separation Point.  
University of Bristol, Department of Aeronautical Engineering.  
Report RKN/7802.
- Nangia, R.K. Secondary Separation Modelling on Slender Wings.  
University of Bristol, Department of Aeronautical Engineering.  
Report RKN/7901.
- Nangia, R.K. Separated Flow Past Thin Slender Wings of Arbitrary Planform and Spanwise Camber.  
University of Bristol, Department of Aeronautical Engineering.  
Report RKN/7902.
- Nangia, R.K. Leading Edge Devices, Slats on Swept-Back Slender Wings with Flow Separation.  
University of Bristol, Department of Aeronautical Engineering.  
Report RKN/8001 (1980).

With best wishes,  
Yours sincerely,

  
Dr R K Nangia

APPENDIX B

ICAS PAPER NO. 14.2

THE 12TH CONGRESS OF THE  
INTERNATIONAL COUNCIL OF THE AERONAUTICAL SCIENCES

OCTOBER 12-17, 1980

NUMERICAL PREDICTION OF VORTEX CORES FROM THE LEADING  
AND TRAILING EDGES OF DELTA WINGS

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MUNICH, FEDERAL REPUBLIC OF GERMANY

# Numerical Prediction of Vortex Cores of the Leading and Trailing Edges of Delta Wings\*

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## **ABSTRACT**

The purpose of the present paper is to predict the roll-up of the vortex sheets emanating from the leading- and trailing-edges of delta wings with emphasis on the interaction of vortex cores beyond the trailing edge. The motivation behind the present work is the recent experimental data published by Hummel.

The Nonlinear Discrete-Vortex method (NDV-method) is modified and extended to predict the leading- and trailing-vortex cores beyond the trailing edge. The present model alleviates the problems previously encountered in predicting satisfactory pressure distributions. This is accomplished by lumping the free-vortex lines during the iteration procedure. The leading- and trailing-edge cores and their feeding sheets are obtained as parts of the solution.

The numerical results show that the NDV-method is successful in confirming the formation of a trailing-edge core with opposite circulation and opposite roll-up to those of the leading-edge core. This work is a break through in the high angle of attack aerodynamics and moreover, it is the first numerical prediction done on this problem.

## **NOMENCLATURE**

|                         |   |
|-------------------------|---|
| AR                      | wing aspect ratio   |
| b                       | wing half span  |
| b(x)                    | local half span   |
| C <sub>p</sub>          | static pressure   |
| ΔC <sub>p</sub>         | net surface pressure  |
| C <sub>r</sub>          | wing root chord   |
| LE                      | leading edge  |
| LEC                     | leading-edge core   |
| TE                      | trailing edge   |
| TEC                     | trailing-edge core  |
| xyz                     | wing-fixed coordinates, origin at wing nose, x-axis is along the root chord, y-axis is perpendicular to wing                                  |
| $\bar{x}\bar{y}\bar{z}$ | wind-fixed coordinates, origin at the trailing edge, $\bar{x}$ -axis is parallel to the free stream direction                                 |
| $v_x, v_y, v_z$         | components of $\vec{v}$ in the wind coordinate system   |
| $v_{\bar{x}\bar{x}}$    | component of $\vec{v}$ in planes $\bar{x} = \text{constant}$ ,<br>$(v_{\bar{x}\bar{x}} = \sqrt{v_{\bar{y}\bar{y}}^2 + v_{\bar{z}\bar{z}}^2})$ |
| $\alpha$                | angle of attack   |
| $\xi, \eta, \zeta$      | dimensionless wind-fixed coordinates,<br>$\xi = \bar{x}/b, \eta = \bar{y}/b, \zeta = \bar{z}/b$   |

## **1 DESCRIPTION OF THE FLOW FIELD**

The flow field around highly swept wings at moderate to large angles of attack is characterized by flow separations from the leading and side

edges due to strong cross flows. The flows from the upper and lower surfaces of the wing leave at these edges forming free-shear layers. The separated free-shear layers roll up spirally and form two vortex cores which are continuously fed by the vorticity shed from the attached boundary layers on the wing surfaces through the free-shear layers. This flow separation is known as the "primary separation." It has a dominant effect on the aerodynamic characteristics due to the large strength of its vortex core and its proximity to the upper surface of the wing. It generates a large suction pressure peak on the upper surface under the primary vortex core.<sup>1-3</sup>

The adverse pressure gradient outboard of the suction peaks affects the boundary layer flow on the upper surface and "secondary separation" from the wing surface occurs. The secondary separated flow forms either an additional free-shear layer or a bubble depending on the angle of attack and the wing aspect ratio. In the range of moderate to large angles of attack, the secondary free-shear layer rolls up spirally in an opposite sense to that of the primary free-shear layer and forms a secondary vortex core with a strength much smaller than that of the primary core and of opposite strength.<sup>1-5</sup>

The effect of secondary separation on the upper-surface pressure distribution depends on the type of boundary-layer flow on the upper surface. For a laminar boundary layer, the secondary core produces another lower pressure peak between the secondary and primary lines of separation. For a turbulent boundary layer, such a pressure peak is hardly noticed, and the pressure peak corresponding to the primary separation is higher than that of the laminar boundary-layer flow.<sup>5</sup> Therefore, when inviscid models are used to predict the pressure distribution, the calculated results should be compared with the experimental data corresponding to a turbulent boundary-layer flow.<sup>6</sup>

A third type of flow involving a "tertiary" separation may occur between the lines of secondary and primary separation due to an adverse pressure gradient created by the secondary vortex core.

The free-shear layer emanating from the trailing edge of a wing with highly swept-back leading edge is of opposite strength to that of the primary free-shear layer. Beyond the trailing edge, it rolls up spirally in an opposite sense to that of the primary free-shear layer and forms a vortex core. The trailing core has the same sense of strength and core rotation as the secondary vortex core, although they originate from different phenomena. However, their interaction beyond the trailing edge is still unknown.<sup>5,7</sup> Figure 1 (reproduced from Reference 5) shows a schematic of

\*This research work is sponsored by NASA Langley Research Center under NSG 15-0, Dr. E. Carson Yates, Jr. is the technical monitor.  
Associate Professor, AIAA member

the vortex formation behind a slender delta wing. Figure 1-b shows that the secondary vortex sheet and the trailing-edge vortex sheet have the same sense of rotation and the same spanwise location. Their sense of rotation is opposite to that of the primary vortex sheet.

The purpose of the present paper is to predict the roll up of the free-vortex sheets emanating from the leading and trailing edges of delta wings. The main emphasis is to confirm the measurements and conclusions of Hummel<sup>5</sup> that the trailing-edge sheet rolls up into a vortex core within a distance of  $1/4$  root chord from the trailing edge, that its rotation is opposite to that of the primary vortex sheet, and that the trailing edge core is of origin different from the secondary-vortex core although they have the same sense of rotation.

## II. EXISTING MATHEMATICAL MODELS

The description given above shows that the flow field contains four different vortex cores, primary, secondary, tertiary, and trailing-edge vortex cores. The secondary and tertiary cores are due to viscous phenomena and cannot be modeled by using inviscid models. However, their effects are small particularly when the turbulent boundary-layer flow exists on the upper surface of the wing. Hence, they are neglected.

In most analyses, the attached boundary layers on the wing surfaces are represented by bound-vortex sheets while the separated free-shear layers are modeled by free-vortex sheets. The free-vortex sheets join the bound-vortex sheets along the separation lines which are known a priori for wings with sharp edges. Moreover, we assume that vortex-breakdown points are far downstream so that the primary core size and its variation in the vicinity of the wing are neglected. In fact, this assumption limits the large angles of attack at which the inviscid model is applicable. Furthermore, the flow outside the bound-vortex sheet(s) and the free-vortex sheet(s) can be assumed irrotational.

Within these assumptions, the resulting potential flow model represents the main features of the real flow to a high degree of accuracy.

The literature contains several steady and unsteady inviscid-flow models with various degrees of limitations and drawbacks.

The first group of models uses slender-body theory and conical flow assumption.<sup>10-18</sup> These models satisfactorily predict the pressure distribution over the front portion of the wing surface. In the rear portion, the models fail to predict a satisfactory pressure distribution because Kutta condition is violated at the trailing edge. Such models are limited to simple delta planforms.

The second group of models uses a nonlinear discrete-vortex method.<sup>19-31</sup> In these models, the bound-vortex sheet and the free-vortex sheets are approximated by a set of concentrated vortex lines. The bound-vortex sheet is replaced by a bound-vortex lattice, while the free-vortex sheet is

replaced by segmented free-vortex lines (in the case of steady flow) or by a growing free-vortex lattice (in the case of unsteady flow). The boundary conditions are satisfied at certain control points on the bound- and free-vortex system using an iterative technique. Excellent agreement was found<sup>21,23</sup> between calculated and experimental total aerodynamic characteristics, and the agreement between calculated and experimental section characteristics was satisfactory for wings with only side-edge separation. For wings with leading-edge separations, however, the agreement was less than satisfactory for some cases.

Although the discrete-vortex model has for many years worked very well for attached-flow problems<sup>32</sup>, when vortex-type separation from leading edges and/or tips occurs, the free-vortex system lies close to the lifting-surface bound vortices and results are found to be sensitive to variations in the shapes of the quadrilateral vortex elements and the relative lengths or the vortex segments.<sup>33</sup> The best agreement was obtained by using the vortex arrangement developed in reference 21.

The third group of models employs doublet-panels.<sup>34-37</sup> In this method, the wing and its free-vortex sheets are divided into networks of quadrilateral panels. Each panel of the networks representing the wing has a biquadratic local doublet distribution and a bilinear local source distribution. The panels of networks representing the free-vortex sheets have biquadratic local doublet distributions. Source and doublet splines are used to express the distributions of singularities on the networks in terms of discrete values of singularity strength at certain standard points on each network. The boundary conditions and continuity of singularity strengths across abutting networks are enforced at certain standard points on each network. The results of this method are generally good when the solution converges.<sup>38,39</sup> Apparently, the difficulty in obtaining convergence is due to the failure in satisfying the continuity of the derivatives of the doublet strength across abutting networks. This is equivalent to the existence of concentrated vortex lines between abutting networks.

The doublet panel method was extended<sup>40</sup> to include the effect of entrainment of the primary vortex cores through an empirical approach. The results indicated that the entrainment increased the normal-force coefficient substantially over the experimental values.

An excellent review of the inviscid models discussed above is given in reference 41. Reference 42 evaluates the AGARD Symposium on High Angle of Attack Aerodynamics, where many related works are discussed.

The fourth group of models employs a nonlinear hybrid vortex method.<sup>43,44</sup> In this method, continuous vorticity and vortex-line representations of the wing and its separated free-shear layers are used. Continuous vorticity is used in the near-field calculations while discrete vortex-lines are used in the far-field calculations.

The wing and its free-shear layers are divided into quadrilateral vortex panels having first-order vorticity distributions. The aerodynamic boundary conditions and continuity of the vorticity distributions are satisfied at certain nodal points on the vortex panels. An iterative technique is used to satisfy these conditions in order to obtain the vorticity distribution and the shapes of the free-vortex sheets. The calculated net surface pressure and section normal-force coefficients for wings with tip separation are in good agreement with the experimental data and the nonlinear discrete-vortex method. Work is underway to include wings with leading-edge separation and to improve flow modeling by using higher-order vorticity distribution and nonplanar panels.

So far, none of the four groups has been used to predict the leading- and trailing-edge vortex cores. The present work is the first one in which such a prediction is considered and successfully accomplish this goal.

### III. THE EXISTING NONLINEAR DISCRETE-VORTEX METHOD (NDV-METHOD) AND ITS DRAWBACKS

After the AGARD Symposium on High Angle of Attack Aerodynamics<sup>7</sup>, the first attempt to predict the vortex cores was done by the NDV-method which was developed in reference 21 and was later refined in reference 31. Next, a critical evaluation of the existing model is given in order to pinpoint its drawbacks.

In this model, as well as in all the other existing models of the NDV-method<sup>19-31</sup>, no attempt has been made to model the leading-edge vortex core. The vortex sheets emanating from the leading and trailing edges of a delta wing are replaced by a system of vortex lines. Each vortex line is divided into straight, short vortex segments and a semi-infinite vortex line. The dimensionless length of each vortex segment is equal to a unit length (the chord length of a quadrilateral bound-vortex element is taken as the characteristic length of the model).

Starting with an initial guess<sup>14,15</sup> for the inclination of the free-vortex lines representing the free-vortex sheets, the boundary condition on the bound-vortex lattice representing the bound-vortex sheet are satisfied at certain control points. The resulting set of linear algebraic equations is solved and the circulation distribution is obtained. Next, with the known circulation distribution, the positions of the vortex segments of the free-vortex lines are adjusted in order to satisfy the boundary conditions on the free-vortex sheets. These two steps of calculations represent one iterative cycle. Several iterative cycles are performed until the positions of the vortex segments or the circulation distribution converge.

Figure 2 shows a typical converged solution of the system of free-vortex lines in three views for a delta wing of aspect ratio of unity and 15° angle of attack. The plan view also shows the arrangement of bound-vortex lattice. In the three dimensional view the leading-edge core (LEC) is shown. This core is calculated after the solution converges and it represents the centroid of the

leading-edge vortex system. It is calculated in cross-flow planes.

The comparison of height and spanwise position of the calculated centroid with those of the measured leading-edge core<sup>1,4</sup> is given in figure 3. Although this comparison was encouraging, the calculated centroid does not model the physical vortex core where the vortex core is continuously fed with vorticity from the leading edge through the free-shear layers. Using the system of free-vortex lines, the total-aerodynamic loads were accurately calculated but the calculated surface pressure distribution was unsatisfactory. On the other hand, using the centroid of the leading-edge vortex system instead of the leading-edge vortex system, the calculated surface pressure improved<sup>31</sup> (this does not satisfy the no-penetration condition on the wing since the centroid is calculated after the solution converges.) It should be noted here that replacing the leading-edge vortex system by its centroid is similar to the model used by Legendre<sup>10</sup>.

Figure 4 shows a recent converged solution of the system of free-vortex lines with a long deformed wake. It can be seen that the trace of the trailing-edge vortex system in cross planes indicates that the sheet tends to deform upwards showing a tendency to form a trailing-edge vortex core. However, the cross-flow planes taken further downstream shows that the free-vortex lines leapfrog. This does not represent the real flow. Figure 5 shows four of this cross-flow planes taken perpendicular to the wind direction.

It is clearly seen from the few examples given above, that the existing model of the NDV-method does not realistically model the leading- and trailing-edge vortex cores. Therefore, the model and the numerical technique must be modified in order to obtain realistic vortex cores.

### IV. MODIFICATION OF THE NDV-METHOD AND MODELING OF THE LEADING-EDGE CORE

The most obvious drawback of the model is the lack of a realistic model of the leading-edge core and its feeding vortex sheet. Modeling of the leading-edge vortex core and its feeding sheet had been first introduced by Brown and Michael<sup>11</sup>. However, the leading-edge sheet in their model was taken as a planar surface and hence it does not represent the real flow. Mangler and Smith<sup>12</sup> and Smith<sup>13</sup> introduced the first realistic model of the vortex core and its feeding sheet. Their model follows the slender body theory and hence it does not account for the deformation of the leading-edge vortex sheet in the chordwise direction. The numerical models given in references 34-37 and 40 also do not account for the deformation of the leading-edge vortex sheet in the chordwise direction. Consequently, the leading-edge vortex sheet does not feed the leading-edge vortex core beyond the wing trailing edge. Moreover, the model given in reference 37 represents the free-vortex sheets beyond the trailing-edge by using a prescribed shape which was called a "fixed design wake". Therefore, none of these models are capable of predicting the vortex sheet deformation beyond the trailing edge.

In the present modified model of the NDV-method, all these drawbacks are avoided. In the present method, the old model is used in the first two iterative cycles, this is the coarse model. The centroid of the leading-edge vortex system of the coarse model is calculated and is used as the preliminary leading-edge core. Next, the leading and trailing-edge free-vortex lines are replaced by finer vortex segments, this is the fine model. Then, the iterative cycles proceed. During each iterative cycle, each free-vortex line is allowed to rotate three-dimensionally around the most recently calculated centroid for a prescribed number of turns (1/4, 1/2, 3/4, or 1 turn). Once this is accomplished, the remaining vortex segments of this line is dumped into the centroid. This technique is followed until the circulation distribution converges.

Figure 6 shows typical solutions at different iterative cycles. It can be seen that after one iterative cycle, the system of free-vortex lines of the coarse model shows good deformations. The converged solution, indicated by ITER = 6, shows the leading-edge core and its feeding free-vortex lines. It can also be seen that the free-vortex lines continue to feed the LEC beyond the trailing edge. The trailing-edge core is also indicated on the figure. This will be clearly seen in the cross-flow planes discussed in the next section.

Figures 7 and 8 show converged solutions using a 12 x 12 bound-vortex lattice and different turns of the free-vortex lines.

#### V. NUMERICAL EXAMPLES AND COMPARISONS WITH HUMMEL'S EXPERIMENTAL DATA, PREDICTION OF THE LEC AND TEC

The calculated circulation of the free-vortex lines emanating from the wing trailing-edge was found to be of opposite sign to that of the free-vortex lines emanating from the wing leading-edge. This is consistent with Hummel's measurements. This is not a surprise and it can be explained as follows:

The leading-edge vortex core creates large suction pressure peak and hence the pressure continuously rises in the spanwise direction from the wing axis to the location of the suction peak (this is completely opposite to the spanwise variation of pressure for wings with large aspect ratio and small angles of attack; the classical linearized wing theory.) Consequently, the circulation of the spanwise bound-vortex segments increases in the spanwise direction. In order to satisfy the spatial conservation of circulation at the nodes of the bound-vortex lattice, the spanwise increase of circulation requires the circulation of the chordwise bound-vortex segments to be of opposite sign. In the present model, the leading-edge vortex system originates from spanwise-vortex segments at the leading edge while the trailing-edge vortex system originates from chordwise-vortex segments.

According to the difference in sign of the circulation of the trailing-edge vortex system from that of the leading-edge vortex system, one expects that the trailing-edge vortex system rolls up in an opposite sense to the roll-up of the

leading-edge vortex system.

Figures 9-13 show comparisons between the measured<sup>5</sup> and calculated leading- and trailing-edge sheets and flow directions in cross-flow planes perpendicular to the wind direction. The calculated results are drawn with the same scale as that of Hummel's measurements. The calculated results show a remarkable success of the new model. The predicted sizes and locations of the leading- and trailing-edge vortex sheets are very close to those of measurements. One can also see that the heights and spanwise positions of the LEC are successfully predicted. The sequence of the calculated cross flow planes shows that the roll-up of the trailing-edge sheet occurs in an opposite sense to that of the leading-edge sheet.

Figure 13 shows the formation of the trailing-edge core (TEC). The difference between the predicted and measured positions of the TEC is due to the number of turns taken in calculating the leading-edge vortex system. It is expected to get the best comparison when one full turn is considered. With one full turn of the leading-edge vortex system, the roll up will tighten and larger induced velocities are created which in turn will deform the trailing-edge vortex system upwards and leftwards as well. This refinement is currently tested.

Figure 14 shows a converged solution and the cross flow planes of a delta wing of aspect ratio of 1.45. The formation of the trailing-edge core develops earlier than that of the aspect ratio of unity. Work is underway to test different aspect ratios and different angles of attack.

Figure 15 shows comparisons between the predicted and measured static-pressure contours in different cross flow planes. The predicted sizes, locations, and levels of the pressure contours are in good agreement with those of the measured data.

Figure 16 shows comparisons of the predicted and measured spanwise net surface pressure variation at different chordwise stations. With one full turn of the leading-edge vortex system, the predicted suction pressure peak will reach the measured one particularly in the front portion of the wing due to the proximity of the feeding sheet to the wing surface. Moreover with one full turn, the roll-up tightens and the predicted pressure, away from the location of the suction peak, drops to match the experimental data.

The newly developed computer code is computationally efficient. On a CYBER-175, a typical solution of the 12 x 12 lattice including the calculations of five cross-flow planes takes about 10 minutes of CPU time.

#### CONCLUDING REMARKS

The newly developed NDV-method is a breakthrough in the area of the high angle of attack aerodynamics. It enjoys a remarkable success in predicting, for the first time, the latest experimental data published by Hummel. The present technique cures the problems previously encountered with this method. The computer code which employs the



present technique is highly efficient when compared with the present computer codes. It is the only existing code which is capable of predicting the LEC and TEC beyond the trailing edge. Work is underway to refine the technique by using one full turn of the leading-edge vortex system. This prediction is of paramount importance to the problem of canard-wing configurations at large angles of attack.

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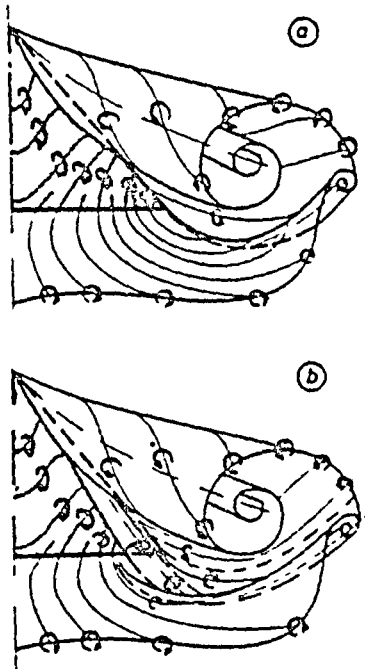


Figure 1. Vortex formation behind a slender delta wing (schematic) a) Elle and Jones (Ref. 8). Hummel and Redeker (Ref. 9) b) Hummel (Ref. 5).

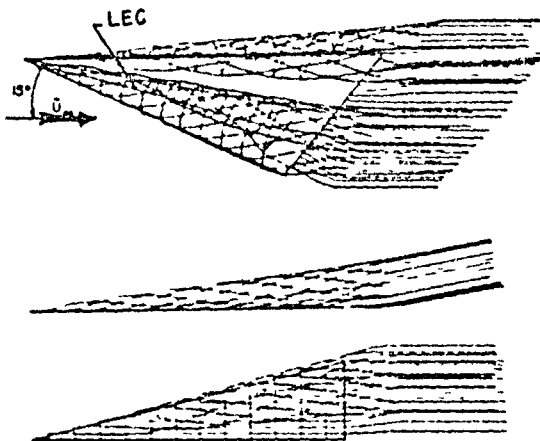


Figure 2. Typical solution of the leading-edge vortex sheet,  $AR = 1$ ,  $12 \times 12$  lattice, Kandil, et.al. (Ref. 31)

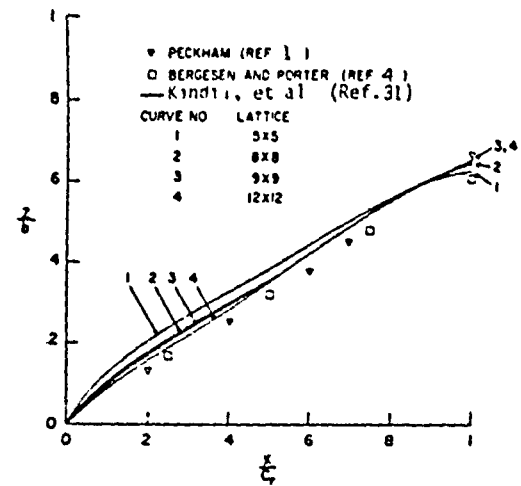
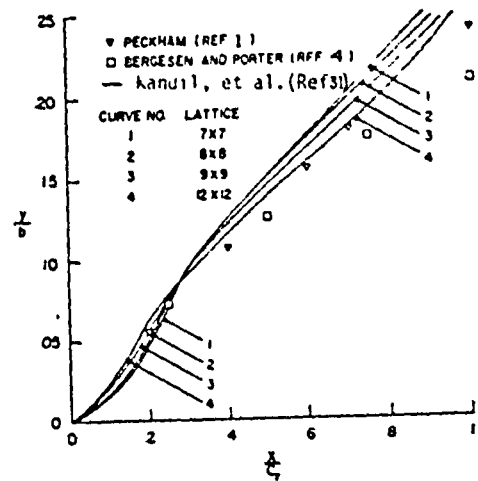


Figure 3. Height and spanwise position of the leading-edge core,  $AR = 1$ ,  $\alpha = 15^\circ$ , Kandil, et.al. (Ref. 31).

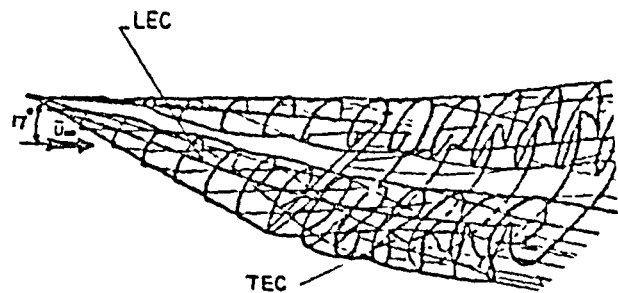


Figure 4. Typical solution of the leading- and trailing-edge vortex sheets,  $AR = 1$ ,  $7 \times 7$  lattice.

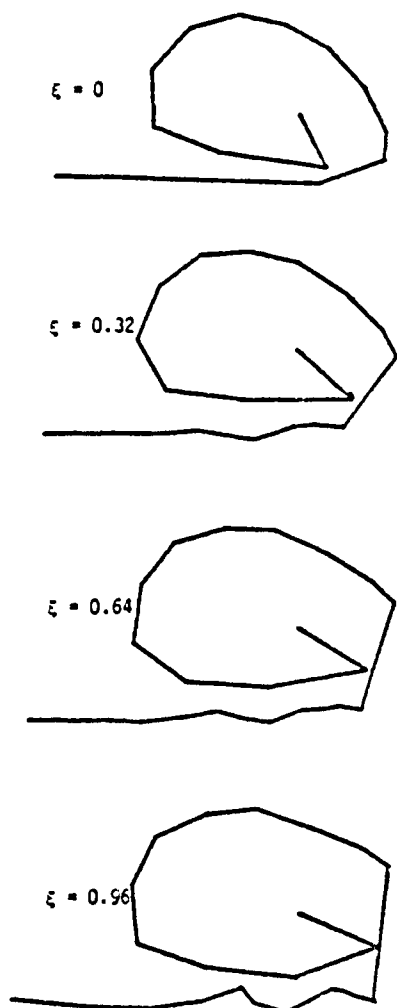


Figure 5. Leading- and trailing-edge sheets behind a delta wing in planes perpendicular to the wind direction,  $AR = 1$ ,  $\alpha = 17^\circ$ ,  $7 \times 7$  lattice.

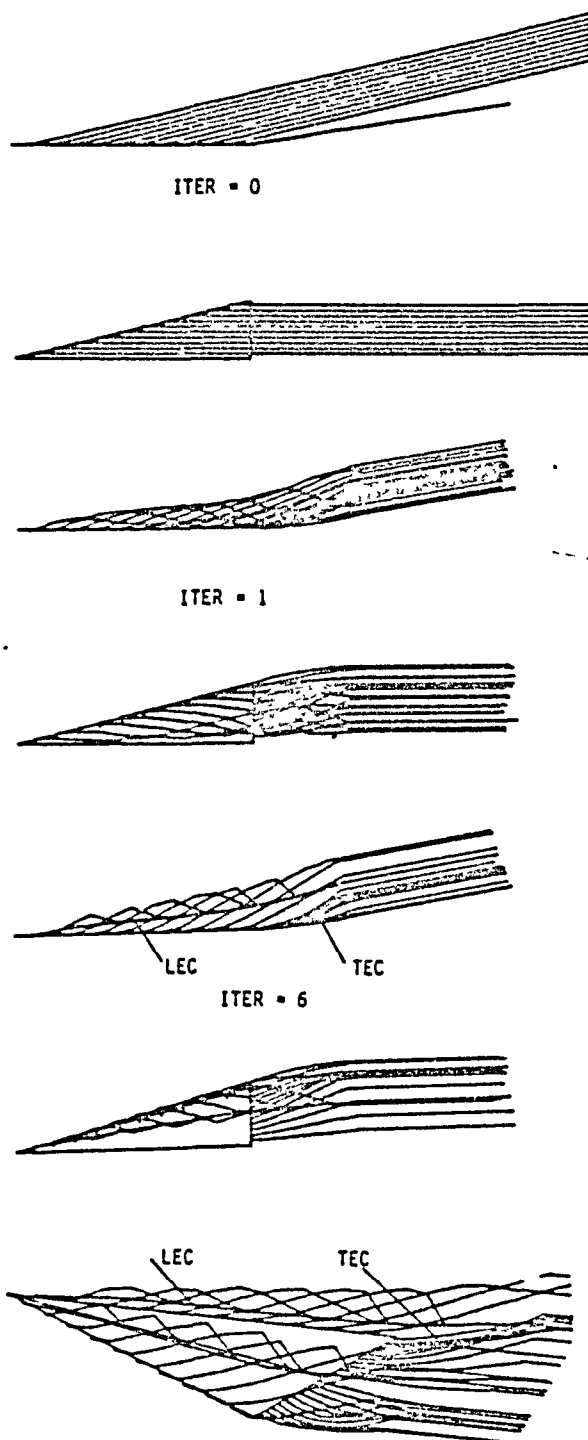


Figure 6. Typical solutions at different iteration steps, ITER = 6 is the converged solution showing the leading- and trailing-edge cores in two- and three-dimensional views,  $AR = 1$ ,  $\alpha = 20.5^\circ$ ,  $10 \times 10$  lattice,  $1/4$  turn.

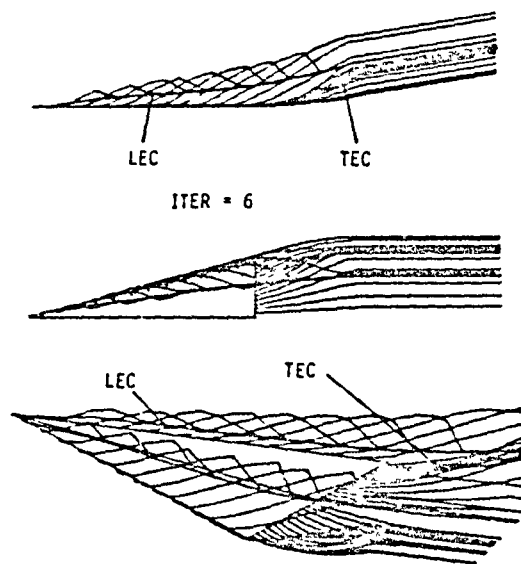


Figure 7 Converged solution showing the leading- and trailing-edge cores in two- and three-dimensional views,  $AR = 1$ ,  $\alpha = 20.5^\circ$ ,  $12 \times 12$  lattice,  $1/4$  turn

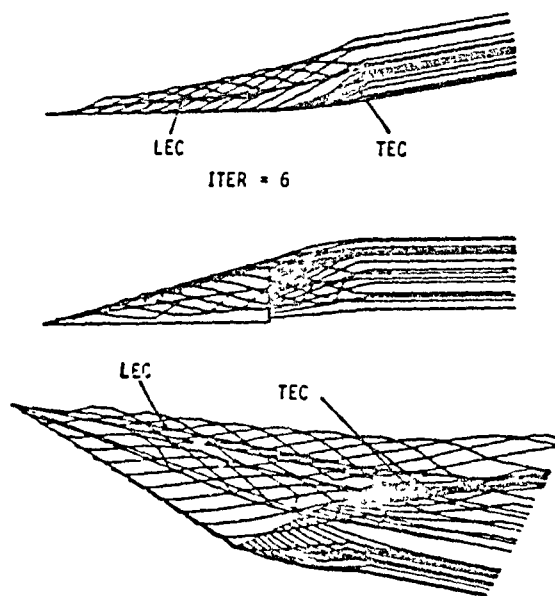


Figure 8 Converged solution showing the leading- and trailing-edge cores in two- and three-dimensional views,  $AR = 1$ ,  $\alpha = 20.5^\circ$ ,  $12 \times 12$  lattice,  $1/2$  turn

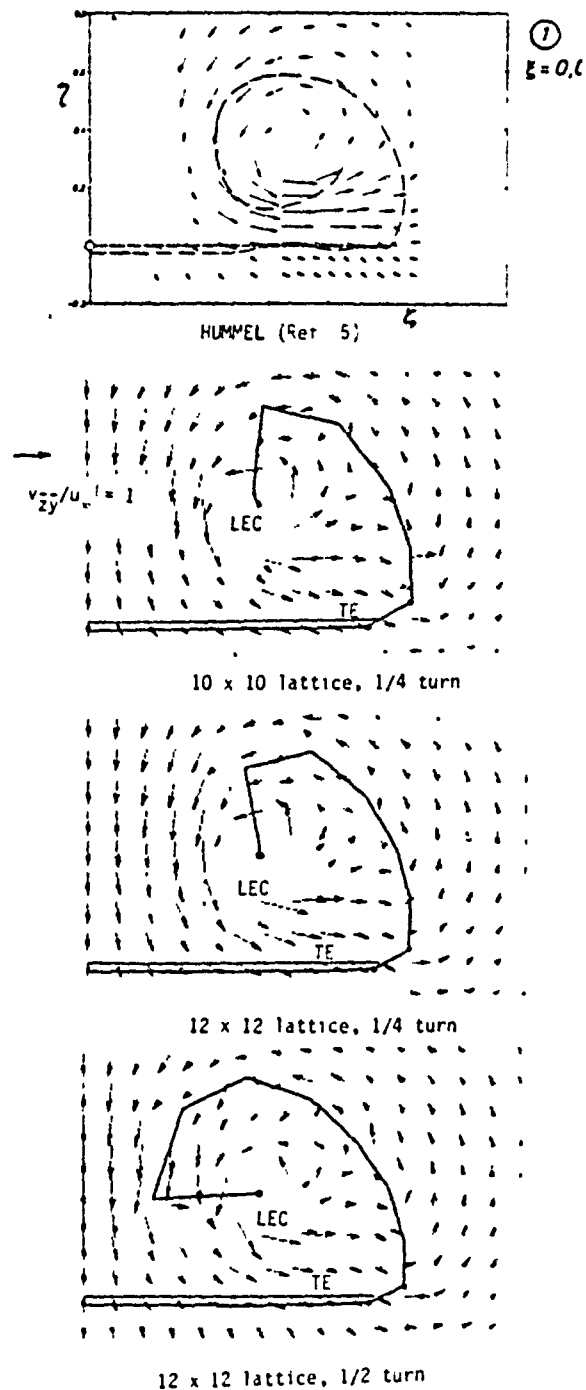


Figure 9 Leading- and trailing-edge sheets and flow direction in a plane perpendicular to the wind direction with different numbers of bound vortex segments and different turns,  $AR = 1$ ,  $\alpha = 20.5^\circ$ ,  $E = 0.08$ .

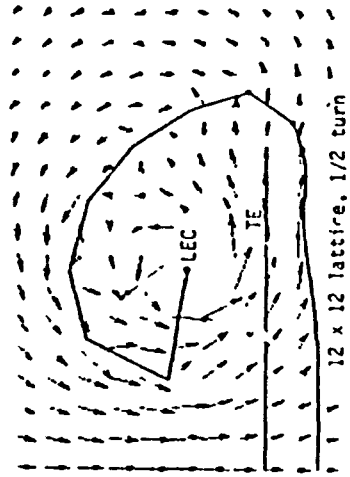
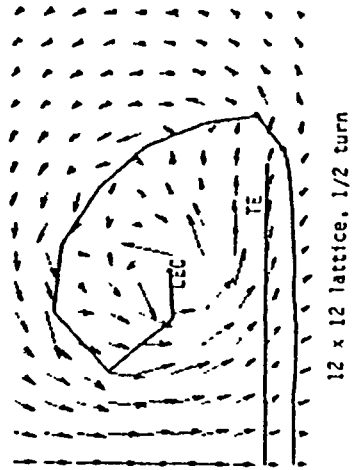
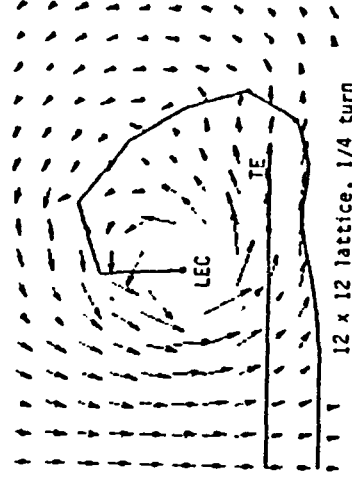
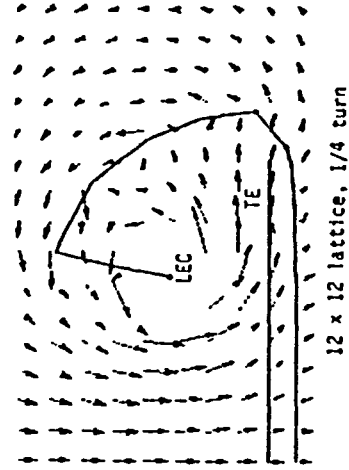
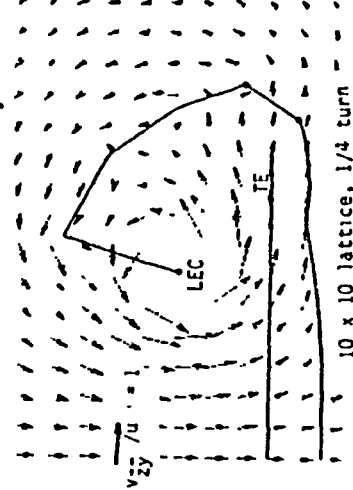
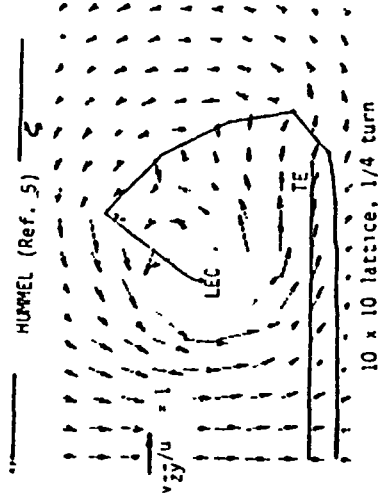
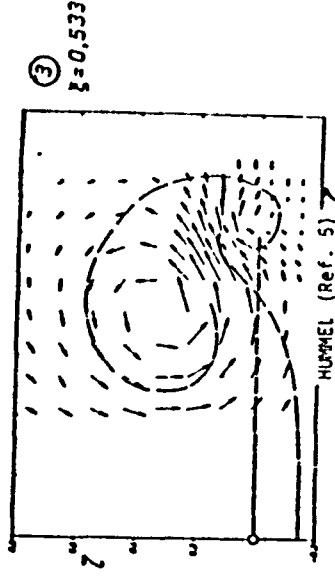
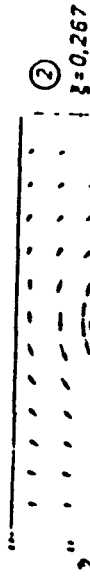
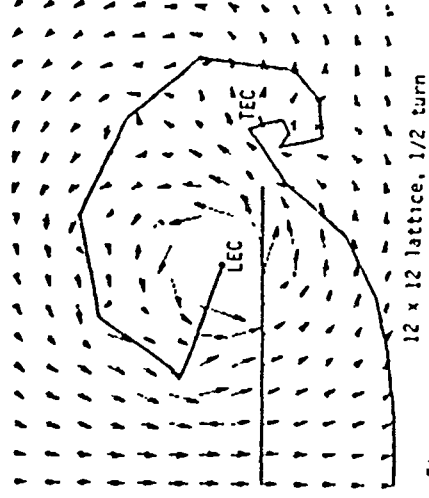
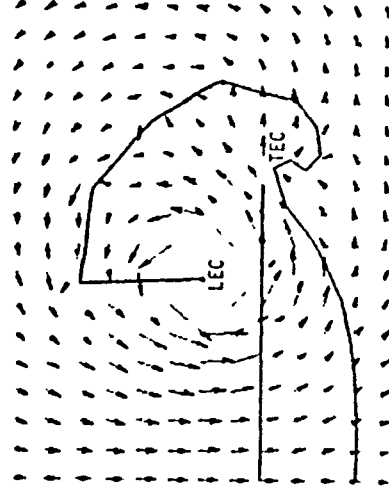
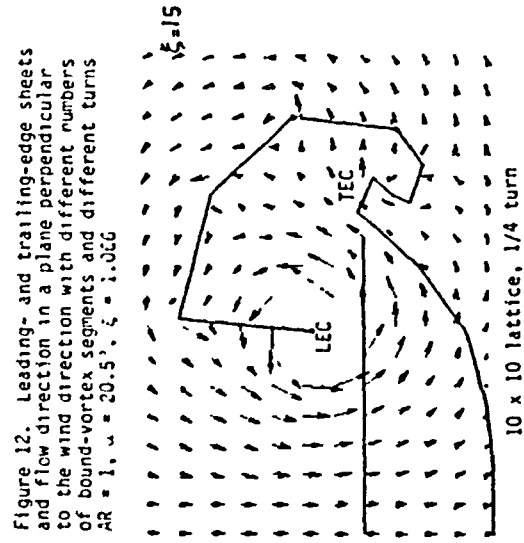
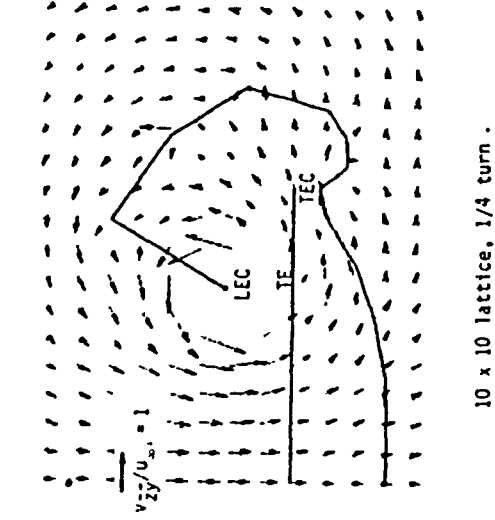
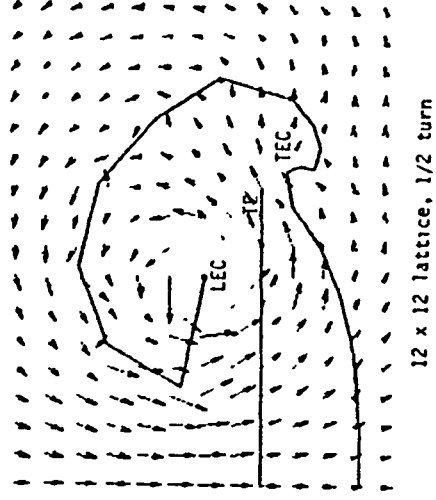
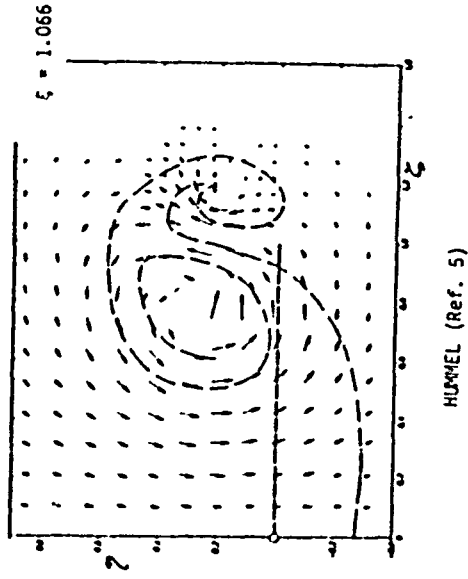


Figure 10. Leading- and trailing-edge sheets and flow direction in a plane perpendicular to the wind direction with different numbers of bound-vortex segments and different turns,  $AR = 1$ ,  $20 \leq \lambda \leq 0.267$ .

Figure 11. Leading- and trailing-edge sheets and flow direction in a plane perpendicular to the wind direction with different numbers of bound-vortex segments and different turns,  $AR = 1$ ,  $20 \leq \lambda \leq 0.533$ .



12 x 12 lattice, 1/8 turn

Figure 13. Leading- and trailing-edge sheets and flow direction in a plane perpendicular to the wind direction with different numbers of bound-vortex segments and different turns.  $AR = 1$ ,  $\alpha = 20.5^\circ$ ,  $\xi = 1.5$ .

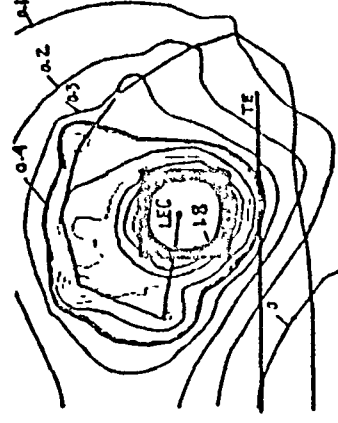
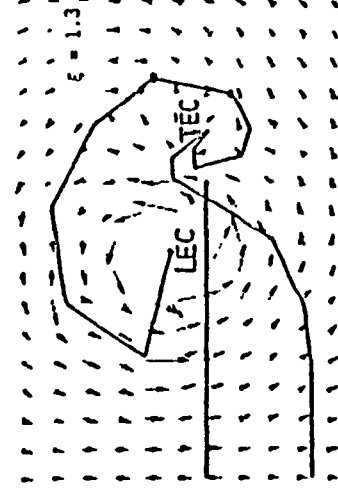
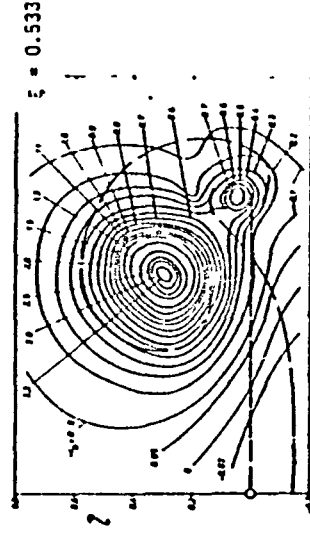
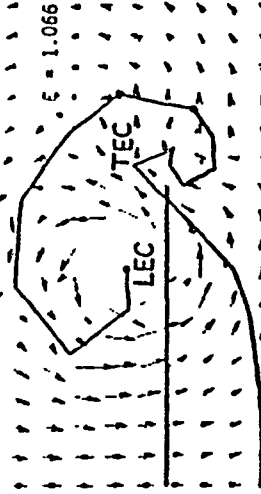
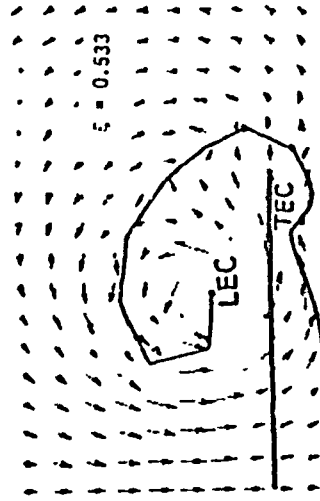
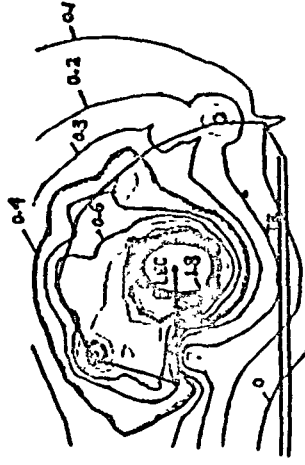
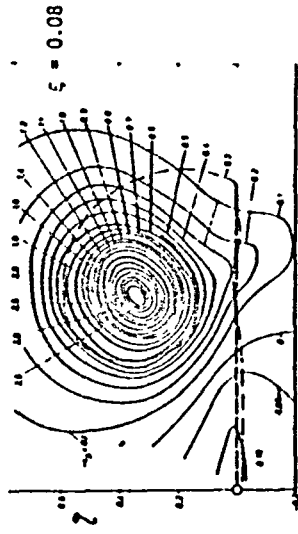
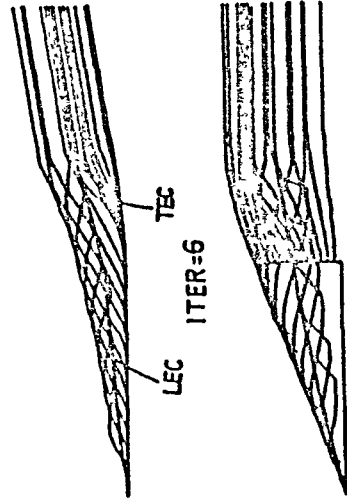


Figure 14. Converged solution, leading- and trailing-edge sheets, and flow direction in different planes perpendicular to the wind direction,  $AR = 1.45$ ,  $\alpha = 20.5^\circ$ ,  $12 \times 12$  lattice,  $1/2$  turn.



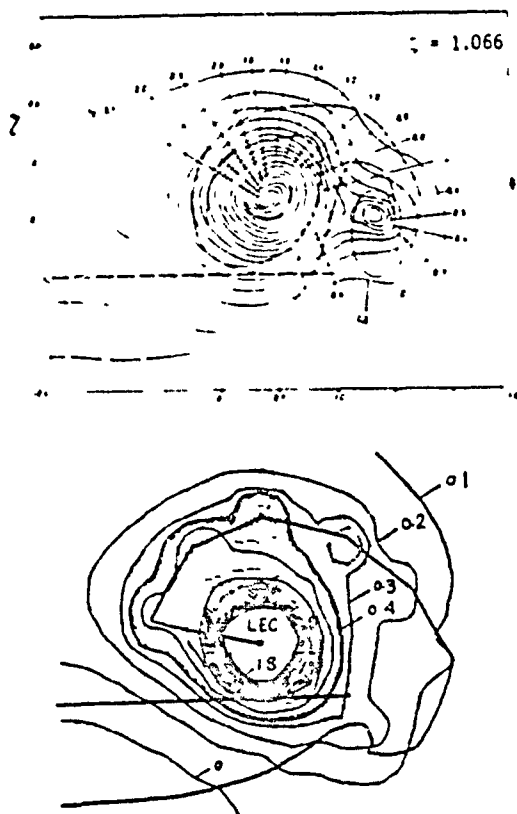


Figure 15. Static pressure contours in different planes perpendicular to the wind direction,  $AR = 1$ ,  $\Delta x = 20.5$ ,  $12 \times 12$  lattice,  $1/2$  turn.

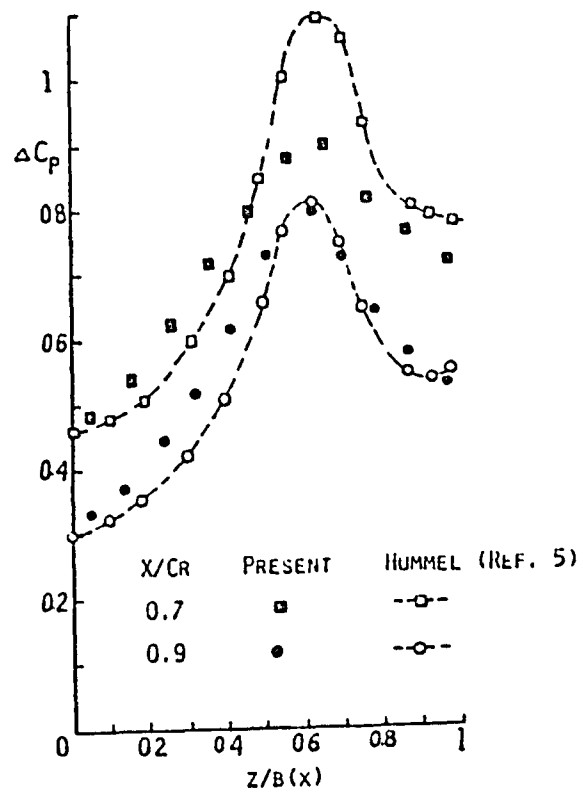


Figure 16. Spanwise net surface pressure variation at different chordwise stations,  $AR = 1$ ,  $\Delta x = 20.5$ ,  $12 \times 12$  lattice,  $1/2$  turn.

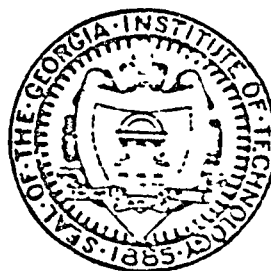
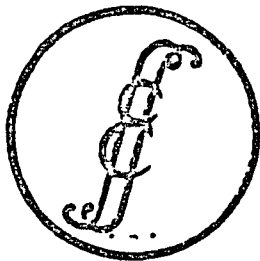
APPENDIX C

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# ABSTRACTS

17th Annual Meeting  
Society of Engineering Science

December 15, 16, and 17, 1980  
Georgia Institute of Technology  
Atlanta, Georgia



Modeling of Vortex Core and Feeding Channels of Delta  
Wings Using the Tailless Discrete Vortex Technique\*

Glenn A. Fowell\*

Department of Mechanical Engineering and Mechanics  
Old Dominion University

A detailed description of the Tailless-Discrete Vortex Approximation, 1-1, which was used for calculating the aerodynamic characteristics of wings with leading-edge separation, is the underlying theory presented between the calculated and experimental pressure distributions. One of the major reasons behind this research is the way in which the separated vortex sheets were modeled by using a system of multiple vortex lines ignoring the vortex cores formed by the separated flow. The present study remedies this problem by looping the vortex lines during the interaction procedure so that the leading-edge trailing-edge vortices (LTC and TEC) and their feeding channels are obtained as parts of the solution. Figure 1 shows the calculated LTC and TEC and their feeding lines for a delta wing. Sections 1 and 2 are normal to the wind direction and their locations are as in the end of the cross section measured from the trailing edge.

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\*This work is supported by NASA grant No. NSG 1569, Dr. E. Charles Moore, Jr. is the technical monitor.

Keywords: Flowfields, VCS Number.

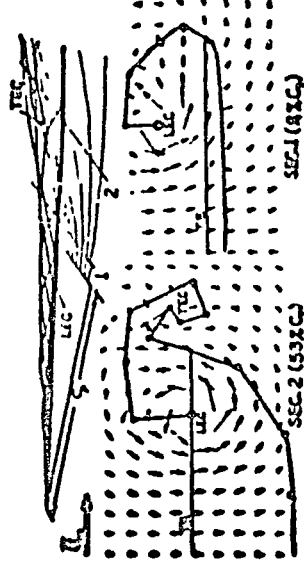


Figure 2. A typical solution of a delta wing (AR=1,  $\alpha=15^\circ$ ) showing the leading-edge trailing-edge vortices, sections 1 & 2 are normal to  $U_\infty$ .

APPENDIX D

Paper No. 81-1263. Recent Improvements in the Prediction of the Leading and Trailing Edge Vortex Cores of Delta Wings. O. A. Kandil, Associate Professor, Department of Mechanical Engineering and Mechanics, Old Dominion Univ., Norfolk, VA.

The recently Modified Nonlinear Discrete Vortex Method (MNDV-Method) has shown a remarkable success in predicting, for the first time, the latest experimental data published by Hummel on vortex formation over a slender delta wing at an angle of attack of  $20.5^\circ$ .

This paper presents the recent developments in the MNDV-Method used to accurately predict the location of the trailing-edge vortex core and the surface pressure distribution. Also, it presents more numerical results of this technique for delta wings with various aspect ratios and various angles of attack in order to study the effects of those two parameters on the formation and interaction of the vortex cores.

Moreover, a viscous core model, based on the boundary-layer-like approximations, is presented. The viscous equations, with outer-edge boundary conditions obtained from the inviscid model, are integrated using a finite-difference marching technique.

APPENDIX E

# ABSTRACT

## A NONLINEAR HYBRID VORTEX METHOD FOR WINGS HAVING SIDE-EDGE SEPARATIONS

Li-Chuan Chu

Old Dominion University, 1980

Chairman: Dr. Osama A. Kandil

A Nonlinear Hybrid-Vortex method (NHV-method) has been developed for predicting the aerodynamic characteristics of wings exhibiting side-edge separations. The present method is a coupling between vortex-panel and vortex-line methods. In the near-field calculations, vortex panels are used while in the far-field calculations, vortex lines are used.

The wing and its free-shear layers are divided into planar quadrilateral panels having first-order vorticity distribution. The aerodynamic boundary conditions and continuity of the vorticity distributions are imposed at certain nodal points on the panels. An iterative technique is used to satisfy these conditions to obtain the vorticity distributions and the wake shape as well. To expedite the convergence of the iterative technique and to avoid excessive distortions of the free-vortex panels, good initial guesses of the shapes of the free-vortex sheets, obtained from the Nonlinear Discrete-Vortex method (NDV method), were used.



This method is used to calculate the distributed and total steady loads on thin flat rectangular wings of different aspect ratios and at different angles of attack. The agreement between the calculated results and the available experimental data is satisfactory.

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